Unit 4 – Proportions and Similar Polygons

Pages 191–192 *Exercises 4.1*

1. No. $\frac{2}{6}$ does not equal $\frac{2+2}{6+2}$; that is, $\frac{2}{6}$ does not equal $\frac{4}{8}$. Also, $\frac{3}{6}$ does not equal $\frac{3-2}{6-2}$; that is, $\frac{3}{6}$ does not equal $\frac{1}{4}$.

2. In each case use the cancellation law to reduce each fraction to lowest terms. Units may be cancelled just as are numerals.

$$\frac{6 \text{ in.}}{8 \text{ in.}} = \frac{3}{4}$$

$$\frac{15 \text{ in.}}{3 \text{ ft.}} = \frac{15 \text{ in.}}{36 \text{ in.}} = \frac{5}{12} \quad 3 \text{ ft.} = 36 \text{ in.}$$

$$\frac{2\frac{1}{2} \text{ in.}}{6\frac{1}{4} \text{ in.}} = \frac{\frac{5}{2} \text{ in.}}{\frac{25}{4} \text{ in.}} = \frac{\cancel{5}^{1}}{\cancel{2}_{1}} \cdot \frac{\cancel{4}^{2}}{\cancel{25}_{5}} = \frac{2}{5}$$

When dividing fractions invert divisor and multiply.

$$\frac{8m}{12m} = \frac{2}{3}$$

3. In effect the line must be divided into two parts $3r$ and $10r$.
Then $3r + 10r = 4\frac{7}{8}$ in., Axiom 6

$13r = \frac{39}{8}$ in., Axiom 8

$r = \frac{1}{\cancel{13}_{1}} \cdot \frac{\cancel{39}^{3}}{8}$ in., Cancellation law

$r = \frac{3}{8}$ in.

Then $3 \cdot r = 3 \cdot \frac{3}{8}$ in. $= \frac{9}{8}$ in. $= 1\frac{1}{8}$ in.
$10 \cdot r = 10 \cdot \frac{3}{8}$ in. $= \frac{15}{4}$ in. $= 3\frac{3}{4}$ in.

4. Solve as any fractional equation.

(a) $\frac{3}{4} = \frac{12}{x}$

$3x = 48$

$x = 16$

(b) $\frac{7}{10} = \frac{x}{5}$

$x = \frac{5 \cdot 7}{10}$

$x = 3.5$

(c) $\frac{6}{5-x} = \frac{7}{3}$

$18 = 7(5 - x)$

$18 = 35 - 7x$

$7x = 35 - 18$

$7x = 17$

$x = \frac{17}{7} = 2\frac{3}{7}$

(d) $\frac{(x-6)}{4} = \frac{4}{5}$

$(x - 6) = \frac{16}{5}$

$x = 3\frac{1}{5} + 6$

$x = 9\frac{1}{5}$

(e) $\frac{m}{n} = \frac{s-x}{r}$

$\frac{mr}{n} = s - x$

$x = \frac{ns - mr}{n}$

(f) $\frac{x-2}{2} = \frac{x}{3x-3}$

$3x^2 - 9x + 6 = 2x$

$3x^2 - 11x + 6 = 0$

$(3x - 2)(x - 3) = 0$

$3x - 2 = 0, x - 3 = 0$

$x = \frac{2}{3}, x = 3$

5. Write each as a fractional equation and "clear of fractions."

(a) $\frac{6}{8} \stackrel{?}{=} \frac{3}{4}$

$24 = 24$

Yes

(b) $\frac{5}{.3} \stackrel{?}{=} \frac{20}{1.2}$

$6 = 6$

Yes

(c) $\frac{2\frac{1}{2}}{4} \stackrel{?}{=} \frac{10}{16}$

$40 = 40$

Yes

(d) $\frac{3 \text{ in.}}{2\frac{1}{2} \text{ ft.}} \stackrel{?}{=} \frac{1 \text{ ft.}}{3\frac{1}{3} \text{ yd.}}$

$\frac{3 \text{ in.}}{30 \text{ in.}} = \frac{1 \text{ ft.}}{10 \text{ ft.}}$

$30 = 30$

Change $2\frac{1}{2}$ ft. to inches and $3\frac{1}{3}$ yd. to feet.

Yes

(e) $\frac{6}{2\frac{5}{8}} \stackrel{?}{=} \frac{16}{7}$

$42 = 42$

Yes

(f) $\frac{8}{.6} \stackrel{?}{=} \frac{5}{.4}$

$3.0 \neq 3.2$

No

6. No. The same unit must be used for the first pair, and the same unit for the second pair; but these two units may be different. For example, see exercise 5(d). Using an inch as the unit for the first two, the ratio is 3:30, that is, 1:10. Using a foot as the unit for the second pair, the ratio is 1:10.

7. (a) $\frac{3}{4} = \frac{9}{x}$

$3x = 36$

$x = 12$

(b) $\frac{4}{3} = \frac{9}{x}$

$4x = 27$

$x = \frac{27}{4}$

$x = 6\frac{3}{4}$

(c) $\frac{3}{9} = \frac{4}{x}$

$3x = 36$

$x = 12$

(d) $\frac{9}{3} = \frac{4}{x}$

$9x = 12$

$x = \frac{12}{9}$

$x = 1\frac{1}{3}$

(e) $\frac{4}{9} = \frac{3}{x}$

$4x = 27$

$x = \frac{27}{4}$

$x = 6\frac{3}{4}$

(f) $\frac{9}{4} = \frac{3}{x}$

$9x = 12$

$x = \frac{12}{9}$

$x = 1\frac{1}{3}$

8. No. The fourth proportional to three given quantities is *not* the same irrespective of the order in which they are taken. Example: $3:4 = 9:x$, $x = 12$; but when the order is changed, $4:9 = 3:x$, $x = 6\frac{3}{4}$.

9. (a) $\frac{5}{x} = \frac{x}{20}$

$x^2 = 100$

$x = \pm\sqrt{100}$

$x = \pm 10$

(b) $\frac{8}{x} = \frac{x}{6}$

$x^2 = 48$

$x = \pm\sqrt{48}$

$x = \pm 4\sqrt{3}$

(c) $\frac{20}{x} = \frac{x}{5}$

$x^2 = 100$

$x = \pm\sqrt{100}$

$x = \pm 10$

(d) $\frac{6}{x} = \frac{x}{8}$

$x^2 = 48$

$x = \pm\sqrt{48}$

$x = \pm 4\sqrt{3}$

(e) $\frac{8}{x} = \frac{x}{18}$

$x^2 = 144$

$x = \pm\sqrt{144}$

$x = \pm 12$

(f) $\frac{a}{x} = \frac{x}{b}$

$x^2 = ab$

$x = \pm\sqrt{ab}$

(g) $\frac{18}{x} = \frac{x}{8}$

$x^2 = 144$

$x = \pm 12$

(h) $\frac{b}{x} = \frac{x}{a}$

$x^2 = ab$

$x = \pm\sqrt{ab}$

10. Yes, for example $\frac{5}{x} = \frac{x}{20}$, $x = \pm 10$ and $\frac{20}{x} = \frac{x}{5}$, $x = \pm 10$.

11. (a) $\frac{3}{6} = \frac{6}{x}$

$3x = 36$

$x = 12$

(b) $\frac{12}{8} = \frac{8}{x}$

$12x = 64$

$x = 5\frac{1}{3}$

(c) $\frac{6}{3} = \frac{3}{x}$

$6x = 9$

$x = \frac{9}{6}$

$x = 1\frac{1}{2}$

(d) $\frac{r}{s} = \frac{s}{x}$

$rx = s^2$

$x = \frac{s^2}{r}$

(e) $\frac{8}{12} = \frac{12}{x}$

$8x = 144$

$x = 18$

(f) $\frac{s}{r} = \frac{r}{x}$

$sx = r^2$

$x = \frac{r^2}{s}$

12. No. The third proportional to two quantities *is not* the same irrespective of the order in which they are taken. Example: See (d) and (f) of exercise 11 above.