12.

$$
\begin{array}{r|l}
 & 5x^2 + x + 2 \\
\hline
x^2 - 1 & 5x^4 + x^3 - 3x^2 - x - 2 \\
 & \underline{5x^4 \qquad - 5x^2} \\
 & \quad x^3 + 2x^2 - x \\
 & \quad \underline{x^3 \qquad - x} \\
 & \qquad 2x^2 \qquad - 2 \\
 & \qquad \underline{2x^2 \qquad - 2}
\end{array}
$$

13.

$$
\begin{array}{r|l}
 & 5x^3 + 4x^2 + 3x + 2 \\
\hline
5x^2 - 4x & 25x^5 + 0x^4 - x^3 - 2x^2 - 8x \\
 & \underline{25x^5 - 20x^4} \\
 & \quad 20x^4 - x^3 \\
 & \quad \underline{20x^4 - 16x^3} \\
 & \qquad 15x^3 - 2x^2 \\
 & \qquad \underline{15x^3 - 12x^2} \\
 & \qquad\quad 10x^2 - 8x \\
 & \qquad\quad \underline{10x^2 - 8x}
\end{array}
$$

14.

$$
\begin{array}{r|l}
 & a^7 - a^6 + 2a^5 - 2a^4 + 3a^3 - 3a^2 + 3a + \frac{-1}{a+1} \\
\hline
a + 1 & a^8 + 0a^7 + a^6 + 0a^5 + a^4 + 0a^3 + 0a^2 + 3a - 1 \\
 & \underline{a^8 + a^7} \\
 & \quad -a^7 + a^6 \\
 & \quad \underline{-a^7 - a^6} \\
 & \qquad 2a^6 + 0a^5 \\
 & \qquad \underline{2a^6 + 2a^5} \\
 & \qquad\quad -2a^5 + a^4 \\
 & \qquad\quad \underline{-2a^5 - 2a^4} \\
 & \qquad\qquad 3a^4 + 0a^3 \\
 & \qquad\qquad \underline{3a^4 + 3a^3} \\
 & \qquad\qquad\quad -3a^3 + 0a^2 \\
 & \qquad\qquad\quad \underline{-3a^3 - 3a^2} \\
 & \qquad\qquad\qquad 3a^2 + 3a \\
 & \qquad\qquad\qquad \underline{3a^2 + 3a} \\
 & \qquad\qquad\qquad\qquad - 1
\end{array}
$$

Remainder may be shown as $\cdots 3a - \frac{1}{a+1}$

15.

$$
\begin{array}{r|l}
 & b^6 + 2b^3 + b + 1 + \frac{4}{b^3 + 4} \\
\hline
b^3 + 4 & b^9 + 6b^6 + b^4 + 9b^3 + 4b + 8 \\
 & \underline{b^9 + 4b^6} \\
 & \quad 2b^6 + b^4 + 9b^3 \\
 & \quad \underline{2b^6 \qquad + 8b^3} \\
 & \qquad b^4 + b^3 + 4b + 8 \\
 & \qquad \underline{b^4 \qquad\quad + 4b} \\
 & \qquad\quad b^3 \qquad + 8 \\
 & \qquad\quad \underline{b^3 \qquad + 4} \\
 & \qquad\qquad\qquad 4
\end{array}
$$

16.

$$
\begin{array}{r|l}
 & z^4 + z^2 + 2 \\
\hline
4z^2 + 2 & 4z^6 + 6z^4 + 10z^2 + 4 \\
 & \underline{4z^6 + 2z^4} \\
 & \quad 4z^4 \quad 10z^2 \\
 & \quad \underline{4z^4 + \quad 2z^2} \\
 & \qquad\quad 8z^2 + 4 \\
 & \qquad\quad \underline{8z^2 + 4}
\end{array}
$$

17.

$$
\begin{array}{r|l}
 & x^2 - xy + y^2 \\
\hline
x^2 + y^2 & x^4 - x^3y + 2x^2y^2 - xy^3 + y^4 \\
 & \underline{x^4 \qquad + x^2y^2} \\
 & \quad - x^3y + x^2y^2 - xy^3 \\
 & \quad \underline{- x^3y \qquad\quad - xy^3} \\
 & \qquad\quad x^2y^2 \qquad + y^4 \\
 & \qquad\quad \underline{x^2y^2 \qquad + y^4}
\end{array}
$$

18.

$$
\begin{array}{r|l}
 & 4m^2 + 2mn + n^2 \\
\hline
2m + n & 8m^3 + 8m^2n + 4mn^2 + n^3 \\
 & \underline{- 8m^3 + 4m^2n} \\
 & \quad 4m^2n + 4mn^2 \\
 & \quad \underline{- 4m^2n + 2mn^2} \\
 & \qquad\quad 2mn^2 + n^3 \\
 & \qquad\quad \underline{2mn^2 + n^3}
\end{array}
$$

19.
$$
\begin{array}{r|l}
 & a^2 - 2a + 4 \\
\hline
a^2 + 2a + 4 & a^4 + 0a^3 + 4a^2 + 0a + 16 \\
 & -\ a^4 + 2a^3 + 4a^2 \\
\hline
 & -\ -2a^3 + 0a^2 + 0a \\
 & \quad -2a^3 - 4a^2 - 8a \\
\hline
 & \qquad -\ 4a^2 + 8a + 16 \\
 & \qquad\quad 4a^2 + 8a + 16 \\
\hline
\end{array}
$$

20.
$$
\begin{array}{r|l}
 & x^3 + 2x^2 + 7x + 20 \\
\hline
x^2 - 2x - 3 & x^5 + 0x^4 + 0x^3 + 0x^2 - 61x - 60 \\
 & -\ x^5 - 2x^4 - 3x^3 \\
\hline
 & -\ 2x^4 + 3x^3 + 0x^2 \\
 & \quad 2x^4 - 4x^3 - 6x^2 \\
\hline
 & \qquad -\ 7x^3 + 6x^2 - 61x \\
 & \qquad\quad 7x^3 - 14x^2 - 21x \\
\hline
 & \qquad\qquad -\ 20x^2 - 40x - 60 \\
 & \qquad\qquad\quad 20x^2 - 40x - 60 \\
\hline
\end{array}
$$

21.
$$
\begin{array}{r|l}
 & a^3 - 4a^2 + 11a - 24 \\
\hline
a^2 + 4a + 5 & a^5 + 0a^4 + 0a^3 + 0a^2 - 41a - 120 \\
 & -\ a^5 + 4a^4 + 5a^3 \\
\hline
 & -4a^4 - 5a^3 + 0a^2 \\
 & -\ -4a^4 - 16a^3 - 20a^2 \\
\hline
 & \qquad -\ 11a^3 + 20a^2 - 41a \\
 & \qquad\quad 11a^3 + 44a^2 + 55a \\
\hline
 & \qquad\qquad -\ -24a^2 - 96a - 120 \\
 & \qquad\qquad\quad -24a^2 - 96a - 120 \\
\hline
\end{array}
$$

22.
$$
\begin{array}{r|l}
 & 5x^3 + 4x^2 + 3x + 2 \\
\hline
5x^2 - 4x & 25x^5 + 0x^4 - x^3 - 2x^2 - 8x \\
 & -\ 25x^5 - 20x^4 \\
\hline
 & -\ 20x^4 - x^3 \\
 & \quad 20x^4 - 16x^3 \\
\hline
 & \qquad -\ 15x^3 - 2x^2 \\
 & \qquad\quad 15x^3 - 12x^2 \\
\hline
 & \qquad\qquad -\ 10x^2 - 8x \\
 & \qquad\qquad\quad 10x^2 - 8x \\
\hline
\end{array}
$$

23.

$$
\begin{array}{r|l}
 & a^7 - a^6 + 2a^5 - 2a^4 + 3a^3 - 3a^2 + 4a - 1 \\
\hline
a + 1 & a^8 + 0a^7 + a^6 + 0a^5 + a^4 + 0a^3 + a^2 + 3a - 1 \\
 & -\ a^8 + a^7 \\
\hline
 & -\ a^7 + a^6 \\
 & -\ a^7 - a^6 \\
\hline
 & 2a^6 + 0a^5 \\
 & -\ 2a^6 + 2a^5 \\
\hline
 & -\ 2a^5 + a^4 \\
 & -\ 2a^5 - 2a^4 \\
\hline
 & 3a^4 + 0a^3 \\
 & -\ 3a^4 + 3a^3 \\
\hline
 & -3a^3 + a^2 \\
 & -3a^3 - 3a^2 \\
\hline
 & 4a^2 + 3a \\
 & -\ 4a^2 + 4a \\
\hline
 & -\ a - 1 \\
 & -\ a - 1 \\
\hline
\end{array}
$$

24.

$$
\begin{array}{r|l}
 & 2y^2 - 3y + 1 \\
\hline
2y^2 + 3y - 1 & 4y^4 + 0y^3 - 9y^2 + 6y - 1 \\
 & -\ 4y^4 + 6y^3 - 2y^2 \\
\hline
 & -\ 6y^3 - 7y^2 + 6y \\
 & -\ 6y^3 - 9y^2 + 3y \\
\hline
 & 2y^2 + 3y - 1 \\
 & -\ 2y^2 + 3y - 1 \\
\hline
\end{array}
$$

25.

$$
\begin{array}{r|l}
 & x^2 - ax - b \\
\hline
ax - b & ax^3 - a^2x^2 - bx^2 + 0x + b^2 \\
 & -\ ax^3 - bx^2 \\
\hline
 & -\ a^2x^2 + 0x \\
 & -\ a^2x^2 + abx \\
\hline
 & -\ abx + b^2 \\
 & -\ abx + b^2 \\
\hline
\end{array}
$$

26.

$$
\begin{array}{r|l}
 & 5x^2 + 2xy - 3y^2 \\
\hline
-5x + 6y & -25x^3 + 20x^2y + 27xy^2 - 18y^3 \\
 & -\ -25x^3 + 30x^2y \\
\hline
 & -10x^2y + 27xy^2 \\
 & -\ -10x^2y + 12xy^2 \\
\hline
 & 15xy^2 - 18y^3 \\
 & -\ 15xy^2 - 18y^3 \\
\hline
\end{array}
$$

27.

$$
\begin{array}{r|l}
 & a^2 - 2ax + x^2 \\
\hline
a^2 - 2ax + x^2 & a^4 - 4a^3x + 6a^2x^2 - 4ax^3 + x^4 \\
 & -\ a^4 - 2a^3x + a^2x^2 \\
\hline
 & -2a^3x + 5a^2x^2 - 4ax^3 \\
 & -\ -2a^3x + 4a^2x^2 - 2ax^3 \\
\hline
 & a^2x^2 - 2ax^3 + x^4 \\
 & -\ a^2x^2 - 2ax^3 + x^4 \\
\hline
\end{array}
$$